SAFE HANDS & IIT-ian's PACE

LEAP TEST# 14 (JEE) ANS KEY Dt. 07-01-2024

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	C	31	D	61	C
2	A	32	A	62	B
3	A	33	D	63	A
4	C	34	B	64	C
5	B	35	A	65	B
6	C	36	A	66	C
7	C	37	A	67	C
8	D	38	D	68	A
9	B	39	D	69	C
10	A	40	A	70	A
11	B	41	A	71	A
12	A	42	C	72	A
13	A	43	C	73	B
14	C	44	C	74	D
15	D	45	A	75	C
16	C	46	B	76	D
17	D	47	C	77	B
18	B	48	A	78	C
19	B	49	C	79	D
20	D	50	B	80	A
21	2.82	51	8	81	1.5
22	6.72	52	40	82	4.8
23	2.5	53	3	83	2.25
24	2	54	6	84	5
25	0.81	55	8	85	1

See Maths solutions on next page.....

SAFE HANDS & PACE

LT-14 (JEE) SOLUTIONS

[redacted]

61)	**c**	**62)**	**b**	**63)**	**a**	**64)**	**c**	**77)**	**b**	**78)**	**c**	**79)**	**d**	**80)**	**a**
65)	**b**	**66)**	**c**	**67)**	**c**	**68)**	**a**	**81)**	**1.5**	**82)**	**4.8**	**83)**	**2.25**	**84)**	**5**
69)	**c**	**70)**	**a**	**71)**	**a**	**72)**	**a**	**85)**	**1**						
73)	**b**	**74)**	**d**	**75)**	**c**	**76)**	**d**								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(c)**

Equation of radical axis (i.e. common chord) of the two circles is

$10x + 4y - a - b = 0$ ...(i)

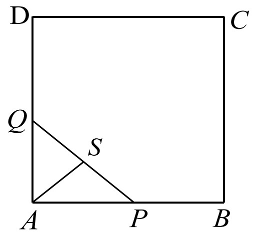
Centre of first circle is $H(-4, -4)$

Since second circle bisects the circumference of the first circle, therefore, centre $H(-4, -4)$ of the first circle must lie on the common chord Eq. (i)

$\therefore\ -40 - 16 - a - b = 0$

$\Rightarrow a + b = -56$

62 **(b)**



Let S be the midpoint of PQ

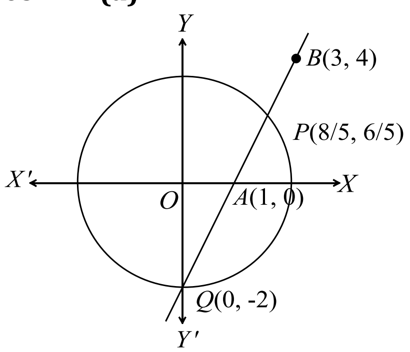
Since $\angle PAQ = \frac{\pi}{2}$, we get $AS = SP = SQ = \frac{1}{2}$

$\Rightarrow S$ lies on the quarter circle of radius $\frac{1}{2}$ with centre at A

Similarly S can also lie on quarter circle of radius $\frac{1}{2}$ with centre at B, C or D

$\Rightarrow$ area $A = 1 - \frac{\pi}{4}$

63 **(a)**



The equation of the line joining $A(1, 0)$ and $B(3, 4)$ is $y = 2x - 2$

This cuts the circle $x^2 + y^2 = 4$ at $Q(0, -2)$ and $P\left(\frac{8}{5}, \frac{6}{5}\right)$

We have $BQ = 3\sqrt{5}, QA = \sqrt{5}, BP = \frac{7}{\sqrt{5}}$ and $PA = \frac{3}{\sqrt{5}}$

$$\therefore\ \alpha = \frac{BP}{PA} = \frac{7/\sqrt{5}}{3/\sqrt{5}} = \frac{7}{3}$$

and $\beta = \frac{BQ}{QA} = \frac{3\sqrt{5}}{-\sqrt{5}} = -3$

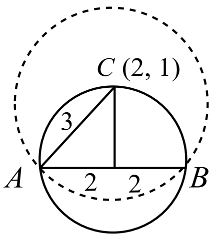
$\therefore\ \alpha, \beta$ are roots of the equation $x^2 - x(\alpha + \beta) + \alpha\beta = 0$

i.e. $x^2 - x\left(\frac{7}{3} - 3\right) + \frac{7}{3}(-3) = 0$

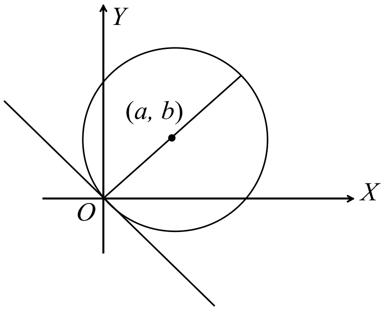
or $3x^2 + 2x - 21 = 0$

64 **(c)**

The centre of given circle is $(1, 3)$ and radius is 2. So, AB is a diameter of the given circle has its mid point as $(1, 3)$. The radius of the required circle is 3



65 **(b)**



Obviously, the slope of the tangent will be – $\left(\frac{1}{b/a}\right)$, i.e., $-\frac{a}{b}$

Hence, the equation of the tangent is $y = -\frac{a}{b}x$, i.e., $by + ax = 0$

66 **(c)**

Equation of any circles through (0, 1) and (0, 6) is

$x^2 + (y-1)(y-6) + \lambda x = 0$

$\Rightarrow x^2 + y^2 + \lambda x - 7y + 6 = 0$

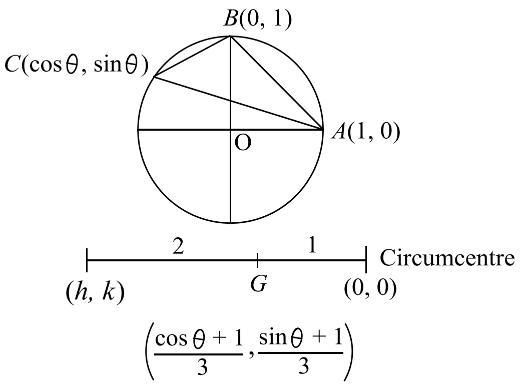
If it touches x-axis, then $x^2 + \lambda x + 6 = 0$ should have equal roots

$\Rightarrow \lambda^2 = 24 \Rightarrow \lambda = \pm\sqrt{24}$

Radius of these circles $= \sqrt{6 + \frac{49}{4} - 6} = \frac{7}{2}$ units

That means we can draw two circles but radius of both circles is $\frac{7}{2}$

67 **(c)**



Let $C\ (\cos\theta, \sin\theta)$; $H(h, k)$ is the orthocenter of the ΔABC

Since circumcentre of the triangle is (0, 0), for orthocenter $h = 1 + \cos\theta$ and $k = 1 + \sin\theta$

Eliminating θ, $(x-1)^2 + (y-1)^2 = 1$

$\therefore\ x^2 + y^2 - 2x - 2y + 1 = 0$

68 **(a)**

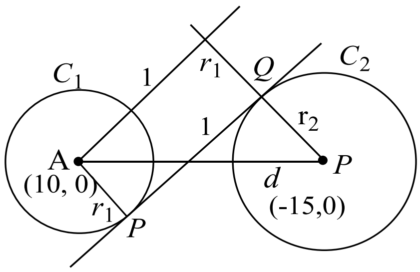
The two normals are $x = 1$ and $y = 2$

Their point of intersection $(1, 2)$ is the centre of the required circle

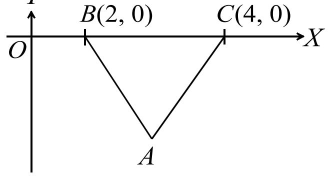
Radius $\frac{|3+8-6|}{5} = 1$

∴ Required circle is
$(x-1)^2+(y-2)^2=1$
i.e. $x^2+y^2-2x-4y+4=0$
69 **(c)**
Substituting $y=mx$ in the equation of circle we get $x^2+m^2x^2=ax+bmx+c=0$ (y/x denotes the slope of the tangent from the origin on the circle)

Since line is touching the circle, we must have discriminant
$\Rightarrow (a+bm)^2-4c(1+m^2)=0$
$\Rightarrow a^2+b^2m^2+2abm-4c-4cm^2=0$
$\Rightarrow m^2(b^2-4c)+2abm+a^2-4c=0$
This equation has two roots m_1 and m_2
$\Rightarrow\ m_1+m_2=-\frac{2ab}{b^2-4c}=\frac{2ab}{4c-b^2}$
70 **(a)**
Centres are (10,0) and $(-15,0)$
and radii are $r_1=6; r_2=9$
Also $d=25$
$r_1+r_2<d$

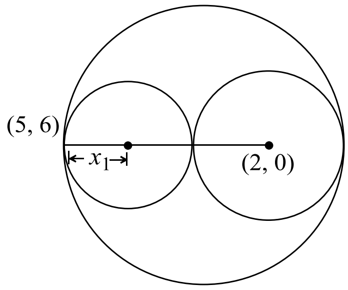


⇒ circles are neither intersecting nor touching
$PQ=\sqrt{d^2-(r_1+r_2)^2}$
$=\sqrt{625-225}$
$=20$
71 **(a)**
If there are more than one rational points on the circumference of the circle $x^2+y^2-2\pi x-2ey+c=0$ (as (π,e) is the centre), then e will be a rational multiple of π, which is not possible. Thus, the number of rational points on the circumference of the circle is at most one
72 **(a)**



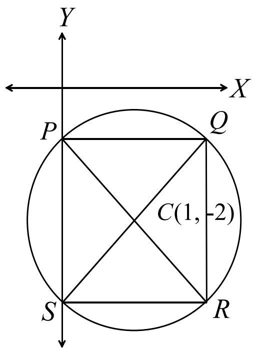
Clearly, $A\equiv(3,-\sqrt{3})$
Centroid of triangle ABC is $\left(3,-\frac{1}{\sqrt{3}}\right)$, thus equation of incircle is
$(x-3)^2+\left(y+\frac{1}{\sqrt{3}}\right)^2=\frac{1}{3}$
$\Rightarrow x^2+y^2-6x+\frac{2y}{\sqrt{3}}+9=0$

73 **(b)**



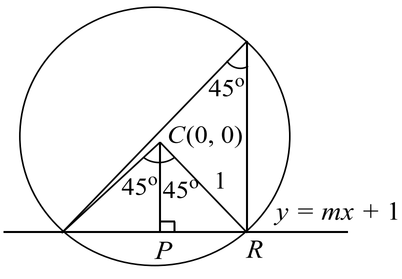
Given circle is $(x-2)^2+y^2=4$
Centre is $(2,0)$ and radius $=2$
Therefore, distance between $(2,0)$ and $(5,6)$ is $\sqrt{9+36}=3\sqrt{5}$
$\Rightarrow r_1=\dfrac{3\sqrt{5}-2}{2}$
and $r_2=\frac{3\sqrt{5}+2}{2}$
$=r_1r_2=\dfrac{41}{4}$

74 **(d)**



Radius of the circle $CQ=\sqrt{2}$
Since $\angle QSR=45°$
Coordinates of Q and S are given by $\left(1\pm\sqrt{2}\cos 45°, -2\pm\sqrt{2}\sin 45°\right)$
or $Q(2,-1)$ and $S(0,-3)$
Coordinates of P and R are given by $\left(1\pm\sqrt{2}\cos 135°, -2\pm\sqrt{2}\sin 135°\right)$
or $P(0,-1)$ and $S(2,-3)$

75 **(c)**



Given circle is $x^2+y^2=1$
$C(0,0)$ and radius $=1$ and chord is $y=mx+1$
$\cos 45°=\dfrac{CP}{CR}$
$CP=$ perpendicular distance from $(0,0)$ to chord $y=mx+1$

$CP = \frac{1}{\sqrt{m^2+1}}$ $(CR = \text{radius} = 1)$

$\Rightarrow \cos 45° = \frac{1/\sqrt{m^2+1}}{1}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{m^2+1}}$

$\Rightarrow m^2 + 1 = 2$

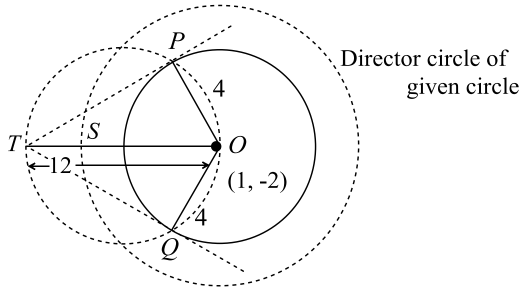
$\Rightarrow m = \neq 1$

76 **(d)**

Given circle $(x-1)^2 + (y+2)^2 = 16$

Its director circle is $(x-1)^2 + (y+2)^2 = 32$

$\Rightarrow OS = 4\sqrt{2}$



Therefore, required distance, $TS = OT - SO = 12 - 4\sqrt{2}$

77 **(b)**

The circle through points of intersection of the two circle $x^2 + y^2 - 6 = 0$ and $x^2 + y^2 - 6x + 8 = 0$ is

$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6x + 8) = 0$

As it passes through (1, 1)

$(1 + 1 - 6) + \lambda(1 + 1 - 6 + 8) = 0$

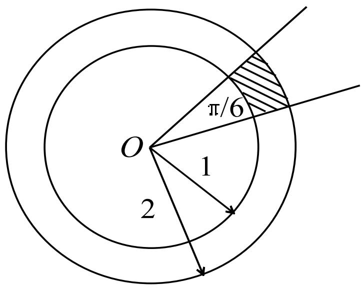
$\Rightarrow \lambda = 1$

$\therefore$ The required circle is

$2x^2 + 2y^2 - 6x + 2 = 0$

or $x^2 + y^2 - 3x + 1 = 0$

79 **(d)**



The angle θ between the lines represented by

$\sqrt{3}\,x^2 - 4xy + \sqrt{3}y^2 = 0$ is given by

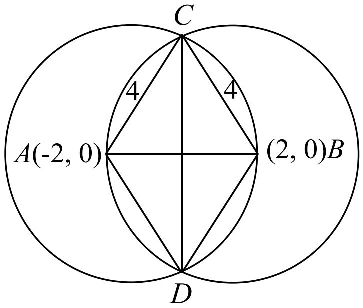
$\theta = \tan^{-1} \frac{\sqrt{h^2 - ab}}{|a+b|}$

$= \tan^{-1} \frac{2\sqrt{2^2 - 3}}{\sqrt{3} + \sqrt{3}} = \frac{1}{\sqrt{3}}$

Gives $\theta = \frac{\pi}{6}$

Hence, the shaded area $= \frac{\pi/6}{2\pi} \times \pi(2^2 - 1^2) = \frac{\pi}{4}$

80 **(a)**



Circles with centre (2, 0) and (−2, 0) each with radius 4

$\Rightarrow$ y-axis is their common chord

ΔABC is equilateral. Hence, area of $ADBC$ is $\frac{2.\sqrt{3}}{4}$ $(4)^2 = 8\sqrt{3}$

Integer Answer Type

81 **(1.5)**

The product of lengths of the perpendicular drawn from any point to the asymptotes of

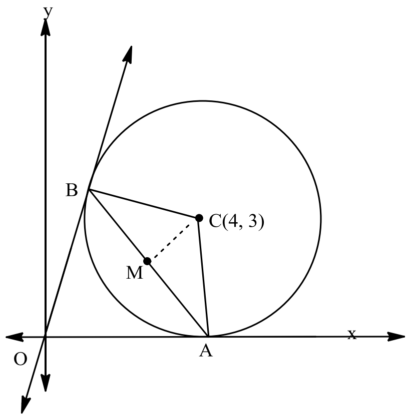
$\frac{x^2}{a^2} - \frac{y^2}{b^2}$ is $\frac{a^2b^2}{a^2+b^2}$

The equation of hyperbola is $\frac{x^2}{2} - \frac{y^2}{1} = 1$

$\Rightarrow$ The reciprocal of the product of lengths of perpendiculars

$= \frac{1}{\frac{(2)(1)}{2+1}} = \frac{3}{2} = 1.5$

82 **(4.8)**



Given equation

$\Leftrightarrow$ $(x-4)^2 + (y-3)^2 = 9$

$\Rightarrow$ Radius = 3

centre = (4,3)

Equation of chord of contact AB is

$\Leftrightarrow$ $x(0) + y(0) - 4(x+0) - 3(y+0) + 16 = 0$

$\Leftrightarrow 4x + 3y - 16 = 0$

$|CM| =$ Perpendicular distance of C from AB

$$= \frac{|4(4) + 3(3) - 16|}{\sqrt{4^2 + 3^2}} = \frac{9}{5}$$

$|BM| = \sqrt{BC^2 - CM^2}$

$$= \sqrt{9 - \frac{81}{25}}$$

$$= \sqrt{\frac{225 - 81}{25}}$$

$$= \sqrt{\frac{144}{25}}$$

$$= \frac{12}{5}$$

$\Rightarrow |AB| = 2|BM| = \frac{24}{5} = 4.8$

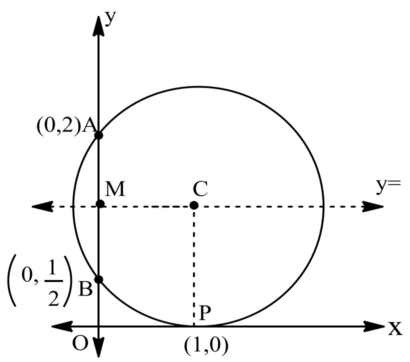
83 **(2.25)**

$f(k, 0) = 0$ has repeated roots $k = 1,1$

$\Rightarrow$ The circle touches the $x -$ axis at $(1,0)$

$f(0, k) = 0$ has roots $\frac{1}{2}, 2$

$\Rightarrow$ The circle passes through $\left(\frac{0,1}{2}\right)$ and $(0,2)$.



$M\left(\frac{0,5}{4}\right)$

$\Rightarrow$ C lies on line $y = \frac{5}{4}$

Also C lies on line $x = 1$

$\Rightarrow$ Centre $C = \left(\frac{1,5}{4}\right)$

$\Rightarrow m = 1, n = \frac{5}{4}$

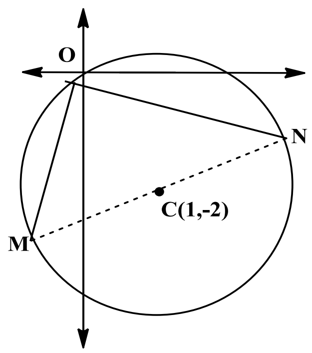
$\Rightarrow m + n = \frac{9}{4} = 2.25$

84 **(5)**

$x^2 + y^2 - 2x + 4y = 0$

$\Leftrightarrow (x-1)^2 + (y+2)^2 = 5$

$\Rightarrow$ Centre $C = (1,-2)$and radius $= \sqrt{5}$



$|MN| = 2\sqrt{5}$

$|OM|^2 + |ON|^2 = 20$

$2|OM|^2 = 20$

$\Rightarrow |OM|^2 = 10$

$\Rightarrow$ Area of $\Delta OMN = \frac{1}{2}|OM|^2 = 5$

85 **(1)**

Let $x + 5 = 14\cos\theta$ and $y - 12 = 14\sin\theta$

$\therefore x^2 + y^2 = (14\cos\theta - 5)^2 + (14\sin\theta + 12)^2$

$= 196 + 25 + 144 + 28\,(12\sin\theta - 5\cos\theta)$

$= 365 + 28\,(12\sin\theta - 5\cos\theta)$

$\therefore \left.\sqrt{x^2+y^2}\right|_{\min} = \sqrt{365 - 28 \times 13} = \sqrt{365 - 364} = 1$